THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

Computational Methods in Financial Mathematics (ME200)

Course duration: 54 hours lecture and class time (Over three weeks)

Summer School Programme Area: Research Methods, Data Science, and Mathematics

LSE Teaching Department: Department of Mathematics

Lead Faculty: Dr Luitgard Veraart (Dept. of Mathematics)

Pre-requisites: Calculus at lower undergraduate level and an introductory course in probability or statistics.

Course Description:

Derivative pricing, investment decisions and financial risk management rely on stochastic models describing financial markets. In these models, quantities of interest such as the price of a financial product often need to be approximated using computational methods. This is a hands-on course in which such methods are introduced and implemented. A particular focus of the course is on Monte Carlo methods, i.e., simulation methods to approximate integrals and expectations of random variables.

Based on the binomial tree model, the fundamental ideas underlying the theory of risk-neutral pricing are introduced. In this context, expressions of option prices as suitable expectations of random variables are derived. It is then shown how these expectations can be evaluated using computational methods.

The course also develops the students' computational skills. Indeed, each time a new computational method is introduced, students implement and apply it to relevant examples during supervised programming sessions. To enable students to do this, the course contains an introduction to programming in R and its applications in financial mathematics.

The course is largely self-contained and reviews the necessary mathematical concepts. No prior programming experience is expected.

Reading:

The main reading material will be the detailed handouts distributed at the beginning of the course. Optionally, the following books might also be helpful:

- J. K. Blitzstein and J. Hwang, Introduction to Probability, Chapman & Hall/CRC, 2015.
- C. P. Robert and G. Casella, Introducing Monte Carlo Methods with R, Springer, 2010.
- S. M. Ross, Simulation, Academic press, 5th edition, 2013.
- R. W. Shonkwiler and F. Mendivil, Explorations in Monte Carlo Methods, Springer, 2009.
- S. E. Shreve, Stochastic Calculus for Finance I: The Binomial Asset Pricing Model, Springer, 2005.

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Course Structure:

- Lectures: 36 hours
- Classes: 18 hours

Assessments:

Formative assessment: Assessed set of homework exercises

Mid-session examination: A mid-session exam (worth 50% of the overall grade) will take place on Tuesday of week two. Results will be released by Monday of week three. The precise time and location of the exam will be circulated during the programme.

Final examination: A final exam (worth 50% of the overall grade) will take place on Friday of week three. Results will be released within two weeks of the exam. The precise time and location of the exam will be circulated during the programme.

Both exams will be closed book exams.

Lecture Schedule:

1. Introduction and overview (Mon 18 Jun)

- Introduction to the Summer School
- Course overview
- What is probability?
- First steps in programming in R

2. Random variables (Tue 19 Jun)

- Introduction to random variables
- Cumulative distribution functions, probability density functions
- Discrete random variables
- Continuous random variables including the Gaussian distribution
- Probability distributions in R

3. Expectation and moments (Wed 20 Jun)

- Expectation, variance and covariance of random variables
- Basic statistics in R

4. Generating random numbers (Thu 21 Jun)

- The linear congruential generator
- The inverse-transform method
- Graphical facilities in R

5. Generating random numbers (cont'd) (Fri 22 Jun)

- Von Neumann's acceptance-rejection method
- Box-Muller method
- Writing new R functions and using control structures in R

6. Monte Carlo estimation (Mon 25 Jun)

- Approximating integrals and expectations via simulation
- Implementation of Monte Carlo methods for several examples in R

Tue 26 Jun - First Exam

7. Variance reduction (Wed 27 Jun)

- Control variates
- Antithetic variates
- Implementation of control and antithetic variates in R

8. Variance reduction (cont'd) (Thu 28 Jun)

- Importance sampling
- Comparison of variance reduction methods and applications in R

9. Introduction to option pricing (Fri 29 Jun)

- The one-period binomial model
- Pricing a European call option in the one-period binomial model
- The concepts of no-arbitrage and replication in the one-period binomial model

10. Option pricing in financial markets with multiple periods (Mon 2 Jul)

- The multi-period binomial model
- Option pricing by replication in the multi-period binomial model
- Computational considerations for the multi-period binomial model and its implementation in R

11. Option pricing in financial markets with multiple periods (cont'd) (Tue 3 Jul)

- The concept of risk-neutral pricing in the multi-period binomial model
- Convergence of the option pricing formula of a European call option in the multi-period binomial model to the Black Scholes option pricing formula of a European call option
- Monte Carlo methods for pricing derivatives

12. Applications and case studies (Wed 4 Jul)

- Pricing of several types of financial derivatives using Monte Carlo methods (with and without variance reduction) and binomial tree approximations.
- Computational considerations and implementations in R

Fri 6 Jul – Second Exam

Content of Seminars:

The seminars will consist of exercises on the mathematical tools developed in the course and computer worksheets where students can develop their programming skills and apply and implement the mathematical methods discussed in the lectures.